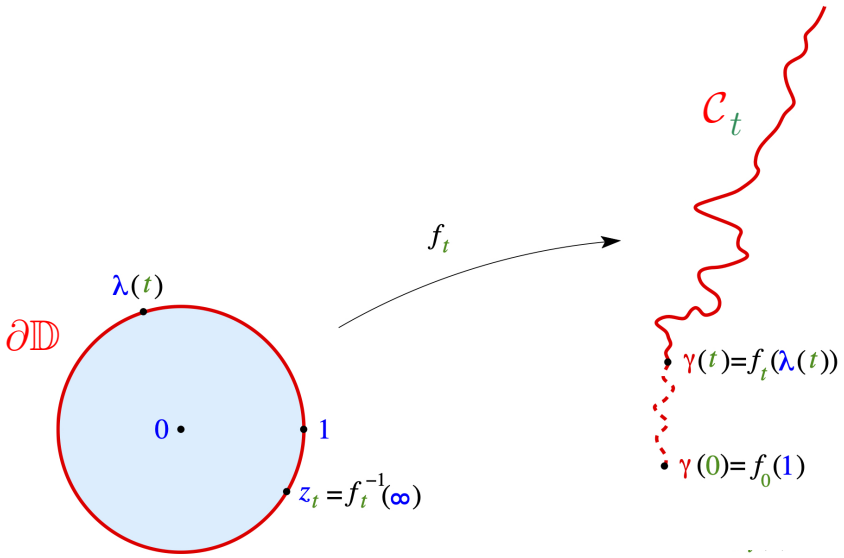
SLE multifractal news

The *Schramm-Loewner evolution* (SLE) is a planar, universal and paradigmatic stochastic process, invented by Oded SCHRAMM in 1999. It led to two Fields Medals, that of Wendelin WERNER in 2006 for the relationship of SLE with planar Brownian motion, and that of Stanislas SMIRNOV in 2010 for its relationship with percolation and the critical Ising model. This process generates universal fractal curves, such as the semi-infinite curve $\mathcal{C}_t$, in **red** in the figure. A random mobile $\lambda(t) = \exp[i(\sqrt{\kappa}B_t + at)]$ travels along the boundary $\partial\mathbb{D}$ of the unit disk $\mathbb{D}$ according to a Brownian motion B_t, with diffusion constant κ and drift constant a. It generates a time-dependent, random conformal transformation $f_t(z \in \mathbb{D})$, which cuts and folds $\partial\mathbb{D}$ on a fractal logarithmic spiral $\mathcal{C}_t$, the image of the mobile being the tip $\gamma(t)$ of the SLE trace. Over time, the process gradually erases $\mathcal{C}_t$.



Bertrand DUPLANTIER from IPhT, in collaboration with Michel ZINSMEISTER from the University of Orléans, a young Chinese mathematician, Yong HAN, and a Vietnamese mathematician, Chi NGUYEN, obtained the **generalized multifractal spectrum** $\beta = \beta(p,q;\kappa,a)$, characterizing the highly singular behavior near the fractal curve of mixed moments, for complex p,q, of f_t and its derivative $f'_t = \partial f_t/\partial z$, such that

$$\int_{r\partial\mathbb{D}} \mathbb{E}\left[\frac{|f'_t(z)^p|}{|f_t(z)^q|}\right] |dz| \asymp (1-r)^{-\beta}, \ r \to 1^-.$$

The so-called **Liouville quantum gravity** method used consists in surveying the complex plane with a quantum measure, where geometric constraints fade like the Cheshire Cat, and in going back to the classical Euclidean plane using the so-called **KPZ** (Knizhnik-Polyakov-Zamolodchikov) relation, first proven in 2011 by BD and Scott SHEFFIELD of MIT. The formula obtained is perfectly verified in two-dimensional integrability loci of $(p,q) \in \mathbb{C}^2$. But to date, there is no purely mathematical proof of this exact result!